Analysis of Observability Constraint on Optimal Feeder Reconfiguration of an Active Distribution Network with μPMUs

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Abstract—Observability is the necessary condition for determining the state variables of the network via state estimation. On the other hand, distribution network reconfiguration (DNR) is an efficient way to reduce the power loss of the distribution network that is fulfilled by changing the open/close state of remotely controlled switches. The study of the impact of observability constraint on the optimal feeder reconfiguration in an active distribution network including uncertain loads and photovoltaics (PVs) is the issue addressed in this paper. There is a measurement infrastructure for observability of the network based on micro phasor measurement units (μPMUs). To perform the study the genetic algorithm (GA) is employed and the simulation are implemented on the benchmark 33-bus distribution network. The simulations revealed that the observability constraint may take the solution of the feeder reconfiguration slightly out of the optimal point when few μPMUs are available.

Keywords—Active distribution network, Micro phasor measurement unit, Observability, Reconfiguration.

I. INTRODUCTION

State estimation (SE) is a general practice in the power networks to obtain the state variables. To execute the SE successfully, the given network should be observable by several measurement devices. Phasor measurement units (PMUs) are accurate measurement devices that improve the real-time monitoring capabilities of the power networks by providing the voltage and current phasor measurements every moment. A new kind of PMU called micro-PMU (μPMU) that is more accurate and less expensive than traditional PMUs is emerging [1]. The advantages of μPMUs over PMUs are the incentives for deploying them in the distribution networks. There are some papers, e.g. [1] that tried to form a fully observable network by deployment of μPMUs. While traditional and pseudo measurements are existent in the distribution networks, designing a stable μPMU-based measurement infrastructure is still important for the modern distribution networks.

The placement of measurement devices is traditionally accomplished for a specific topology even in distribution networks. While, the topology of the distribution networks are
varied occasionally via distribution network reconfiguration (DNR). Consequently, the reconfiguration can influence on the observability of the network. Nevertheless, reconfiguration is incorporated in some recent publications in the area of PMU placement in distribution networks [2,3]. According to [2,3], a great number of PMUs are needed to guarantee the observability of the network under all topologies. However, for some reasons like the cost of PMUs, deploying too many PMUs is not reasonable. In the present paper, it is assumed that μPMUs are installed in the network regardless to the variations of the topology. Therefore, investigating the effect of observability on DNR can be useful for taking better decisions for planning and management of the distribution networks. To the best of authors’ knowledge, none of previous studies concern about the effect of the measurement infrastructure observability of a distribution network on DNR.

The DNR problem is an optimization problem that is commonly limited to some operational and topological constraints. The DNR is done for different purposes, such as minimizing loss, congestion management, etc. In the present work, minimizing loss is considered as the single objective of the reconfiguration problem. Existing solution methods of DNR problem fall into the two main groups; heuristic algorithms and classic optimization techniques, and artificial intelligence based algorithms [4]. Heuristic methods are computationally efficient, but, they may be trapped in the local optima [5]. Classic optimization techniques guarantee the optimality if the model is convex [6]. Artificial intelligence-based methods include artificial neural networks [4] and meta-heuristic algorithms [5,7-10]. Among the introduced methods, meta-heuristics are well-attended in solving the DNR problem. Among the different types of meta-heuristic optimization techniques, genetic algorithms (GAs) are popular and suitable for integer problems, so they are well-fitted for the present work. To incorporate the stochastic nature of distribution networks in the reconfiguration problem, uncertainty of load demands and photovoltaic arrays (PVs) are modeled. The probabilistic reconfiguration is done using probabilistic load flow (PLF) that is based on point estimate method (PEM).

Therefore, in this paper, the impact of observability of the μPMU-based measurement infrastructure on the reconfiguration of a distribution network is investigated. Loss minimization is considered as the objective of the DNR and the new observability constraint is defined in the DNR model. The uncertain loads and PVs modeled by proper probability distribution functions (PDFs) and PLF method based on PEM is deployed for running the probabilistic DNR. The optimization problem is implemented on the benchmark 33-bus distribution network and solved by GA.

The remainder of the paper is constructed as follows: In Section II, formulation of the DNR problem is described considering observability constraint and the uncertainty modeling method is introduced. In Section III, simulation input data is well described and three scenarios are generated and the results of scenarios’ simulation on the test network is presented and discussed. Finally, the conclusions are summarized in Section IV.
The decision variables are the binary open/close state of the branches.

For handling the constraints, the objective function is modified based on a multiplicative penalty method proposed in [14] as expressed in (9).

\[
\varphi = f \times \left( 1 + \frac{1}{5} \sum_{j=1}^{N} \left[ \max \left( 0, 1 - \frac{\alpha_j}{\sigma_j} \right) + \sum_{i=1}^{N} \max \left( 0, 1 - \frac{\beta_j}{\alpha_j} \right) \right]^2 \right) \quad (9)
\]

C. Uncertainty Modeling of PVs and Loads

It is supposed that the historical data of radiation and load demands are available and probabilistic distributions are determined by these data. The Gaussian PDF is widely employed to capture the uncertainty of the load power. The PDF of the active power of the load at bus \( j \) is according to (10) [15].

\[
f_{\text{Load}}(p_{\text{Load}}) = \frac{1}{\sqrt{2\pi}\sigma_{\text{Load}}^j} e^{-\frac{1}{2} \left( \frac{p_{\text{Load}} - \mu_{\text{Load}}^j}{\sigma_{\text{Load}}^j} \right)^2} \quad (10)
\]

There is the same PDF as (10) for reactive power of loads.

The stochastic nature of solar irradiance is widely modeled by Beta PDF. A simple model that relates the output power of PV to the irradiance is as follows [15]:

\[
p_{\text{PV}}^j = K_j \mu \quad (11)
\]

Based on statistic rules, if \( \mu \), and \( \sigma^2 \) are the mean value and variance of the solar irradiance random variable respectively, the mean value and the variance of the output power of PVs will be adjusted as follows:

\[
\mu_{\text{PV}}^j = K_j \mu_j \quad (12)
\]

\[
\sigma_{\text{PV}}^2 = (K_j \mu_j)^2 \sigma_j^2 \quad (13)
\]

The Beta PDF that is defined in [16] for estimating the hourly solar irradiance is used to extract the PDF of output power of the PVs. Having (12) and (13), the shape parameters of Beta distribution, \( \beta \) and \( \alpha \), can be calculated [16].

\[
\beta_j = \left( 1 - \mu_{\text{PV}}^j \right) \left( \frac{1 + \mu_{\text{PV}}^j}{\sigma_{\text{PV}}^j} - 1 \right) \quad (14)
\]

\[
\alpha_j = \frac{\mu_{\text{PV}}^j \beta_j}{1 - \mu_{\text{PV}}^j} \quad (15)
\]

Then the Beta distribution for output power of PVs is generated as (16).

\[
f_b \left( p_{\text{PV}}^j \right) = \frac{\Gamma \left( \alpha_j + \beta_j \right)}{\Gamma \left( \alpha_j \right) \Gamma \left( \beta_j \right)} p_{\text{PV}}^j \left( 1 - p_{\text{PV}}^j \right)^{\beta_j-1} \quad (16)
\]

D. PEM-based Probabilistic Load Flow

PEM method is employed for its lower computational burden than methods like Monte Carlo simulation for running PLF. The formulation is not brought because of space limitation of the paper. In the flowchart of Fig. 1, a summary of proposed reconfiguration method is indicated.

III. SIMULATION AND RESULTS

A. Simulation Inputs

The proposed model is implemented on the benchmark 33-bus distribution network and the optimization problem is solved by GA. The model is simulated by MATLAB R2015b software, MATPOWER 7.0b1 on a personal computer with 2.20 GHz CPU and 8 GB RAM.

The data of benchmark 33-bus distribution network, that is named “base case network”, is taken from [17]. One-line diagram of the network is illustrated in Fig. 2. In Fig. 2, the tie and sectionalizing switches are shown with dashed and solid lines respectively. The active power loss is 202.7 kW in the initial configuration. Two PV arrays are installed at buses 18 and 32. It is assumed that the PVs only inject active power. The mean value of the solar irradiance is supposed to be 400

Fig. 1. The flowchart of observability constrained DNR problem.

Fig. 2. The modified IEEE 33 bus test network.
Regarding to Table II, the performance of proposed model is compared. The CPU time is equal to 10.56 s.

Simulation results of the proposed model and some previous works are compared. The conversion coefficient of the PV installed at bus 18 is 875 and the conversion coefficient of irradiance is considered as 10%.

To do a comprehensive study, different combinations of μPMUs are considered to be located in the network. The number of μPMUs and their corresponding buses are listed in Table I. In all cases, the network is observable by the given number of μPMUs and \( \sigma_j^{\text{min}} \geq 1 \) for all buses, except the last row of Table I in which \( \sigma_j^{\text{min}} \geq 2 \). The μPMU buses are determined for the initial topology of the network and topology changes were ignored.

The GA parameters settings are as follows throughout the simulation: population size: 50; number of iterations: 40; crossover percent: 80%; mutation percent: 40%; mutation rate: 0.4; parents’ selection method: Roulette wheel; and crossover operators: a combination of single-point, double-point, and uniform crossover.

### B. Numerical Results

1) Validation: The Feeder Reconfiguration on the base case network

To validate the proposed model, the base case network is reconfigured by ignoring the observability constraint and the uncertainty associated with the loads. In Table II, the simulation results of the proposed model and some previous works are compared. The CPU time is equal to 10.56 s. Regarding to Table II, the performance of proposed model is similar to the previous works from the aspect of loss reduction percent and the optimal configuration.

2) The probabilistic Reconfiguration on the modified case Network Considering Observability Constraint

In the following, three scenarios are defined to evaluate the impact of the observability constraint on feeder reconfiguration of the modified network. The scenarios are simulated with the different number of μPMUs according to Table I.

#### Scenario 1

\[ \text{Scenario 1: The effect of reconfiguration on observability by reconfiguring the network without considering the observability constraint (} \sigma_j^{\text{min}} = 0 \) \]

#### Scenario 2

\[ \text{Scenario 2: The effect of observability on reconfiguration by reconfiguring the network considering the observability constraint (} \sigma_j^{\text{min}} = 1 \) \]

#### Scenario 3

\[ \text{Scenario 3: The effect of observability on reconfiguration by reconfiguring the network considering the observability constraint (} \sigma_j^{\text{min}} = 2 \) \] and 26 μPMUs.

The scenarios are simulated and the results are listed in Table III. According to Table III, the network loss is declined by 41.3626 kW due to reconfiguration in the scenario 1. Since no restriction on observability is applied, the maximum reduction in loss is occurred in scenario 1. In the scenario 2, when the number of μPMUs are 11, 12, 15 and 18, the amount of reduction in loss is less than the scenario 1 to keep the network observable. As the number of μPMUs increased in the scenario 2, the loss reduction due to reconfiguration became more as well. By deployment of 21 and 26 μPMUs in the scenario 2, the amount of loss reduction became same as the scenario 1. As expected, the observability constraint can

### Table I. The Number and Location of Micro PMUs

<table>
<thead>
<tr>
<th>Number of μPMUs</th>
<th>μPMU Buses</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>2,5,8,11,14,17,21,24,27,30,33</td>
</tr>
<tr>
<td>12</td>
<td>2,5,8,11,14,17,21,23,24,27,30,33</td>
</tr>
<tr>
<td>15</td>
<td>2,5,7,8,11,14,17,18,20,21,23,24,27,30,33</td>
</tr>
<tr>
<td>18</td>
<td>2,5,7,8,11,14,17,18,20,21,23,24,27,29-33</td>
</tr>
<tr>
<td>21</td>
<td>1,2,5,7,8,11,14,16,18,20,21,23,24,26,27,29-33</td>
</tr>
<tr>
<td>26</td>
<td>1,2,4,5,7,8,10,11,13,14,16-18,20-27,29-33</td>
</tr>
</tbody>
</table>

W/m² and is taken from [15] and the coefficient of variations of irradiance is considered as 10%. The conversion coefficient of the PV is 5% for all load demands in the base case network are considered as the mean value of active and reactive power of the loads and the coefficient of variations is considered as 5% for all load points.

### Table II. Validation Results

<table>
<thead>
<tr>
<th>Method</th>
<th>Initial Loss (kW)</th>
<th>Loss after DNR (kW)</th>
<th>Loss Reduction (%)</th>
<th>Open Switches</th>
<th>CPU Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed Method</td>
<td>202.7</td>
<td>139.5513</td>
<td>31.154</td>
<td>s7,s9,s14,s32,s37</td>
<td>-</td>
</tr>
<tr>
<td>MODPSO [8]</td>
<td>202.67</td>
<td>139.53</td>
<td>31.154</td>
<td>s7,s9,s14,s32,s37</td>
<td>-</td>
</tr>
<tr>
<td>EGSA [9]</td>
<td>202.67</td>
<td>139.53</td>
<td>31.154</td>
<td>s7,s9,s14,s32,s37</td>
<td>-</td>
</tr>
<tr>
<td>ACSA [10]</td>
<td>202.68</td>
<td>139.98</td>
<td>30.935</td>
<td>s7,s9,s14,s28,s32</td>
<td>-</td>
</tr>
<tr>
<td>P-PSO [11]</td>
<td>201.87</td>
<td>136.927</td>
<td>32.171</td>
<td>s7,s11,s12,s28,s36</td>
<td>-</td>
</tr>
</tbody>
</table>

### Table III. The Results of the Feeder Reconfiguration for the Modified Network Considering Different Scenarios

<table>
<thead>
<tr>
<th>Scenario 1 (( \sigma_j^{\text{min}} = 0 ))</th>
<th>Mean Value</th>
<th>Standard Deviation</th>
<th>Open Switches</th>
<th>CPU Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before Reconfiguration</td>
<td>138.9432</td>
<td>3.4024</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Scenario 1</td>
<td>97.5806</td>
<td>2.1888</td>
<td>29.77</td>
<td>s7,s9,s14,s30,s37</td>
</tr>
<tr>
<td>Number of μPMUs</td>
<td>11</td>
<td>102.7815</td>
<td>1.9722</td>
<td>26.02</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>102.7815</td>
<td>1.9722</td>
<td>26.02</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>102.3746</td>
<td>1.7264</td>
<td>26.32</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>102.3746</td>
<td>1.7264</td>
<td>26.32</td>
</tr>
<tr>
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<td>21</td>
<td>97.5806</td>
<td>2.1888</td>
<td>29.77</td>
</tr>
<tr>
<td></td>
<td>26</td>
<td>97.5806</td>
<td>2.1888</td>
<td>29.77</td>
</tr>
<tr>
<td>Scenario 2 (( \sigma_j^{\text{min}} = 1 ))</td>
<td>103.0346</td>
<td>1.8070</td>
<td>25.84</td>
<td>s6,s11,s31,s34,s37</td>
</tr>
<tr>
<td>Number of μPMUs</td>
<td>11</td>
<td>102.3746</td>
<td>1.7264</td>
<td>26.32</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>102.3746</td>
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<td>26</td>
<td>97.5806</td>
<td>2.1888</td>
<td>29.77</td>
</tr>
</tbody>
</table>
be ignored if a great number of PMUs are employed; but it becomes economically unaffordable. However, in the worst case that there are only 11 PMUs in the network, the increase of loss is approximately 5.2 kW compared to scenario 1 that can be affordable in exchange for keeping observability. Otherwise, extra PMUs should be installed. In the Scenario 3, despite the existence of the maximum number of PMUs, the loss decreased 35.9 kW, which is 5.454 kW more than the scenario 1, because each bus should be observed at least twice. The obtained loss of scenario 3 is even more than the worst case of scenario 2 while 26 PMUs are deployed.

The observability function value of the buses after executing the scenario 2 and 3 are listed in Table IV which shows that the observability constraint is satisfied in both scenarios in exchange for increasing the loss in some cases.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Observability Function Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 2 ((a_{PMU}^2 = 1))</td>
<td>(O_{2,3,4,5,6,7,8,9,10,11} = 1)</td>
</tr>
<tr>
<td></td>
<td>(O_{3,4,5,6,7,8,9,10,11,12,13,14,15} = 2)</td>
</tr>
<tr>
<td></td>
<td>(O_{3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33} = 3)</td>
</tr>
<tr>
<td></td>
<td>(O_{3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33} = 4)</td>
</tr>
<tr>
<td>Scenario 3 ((a_{PMU}^2 = 2))</td>
<td>(O_{2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32} = 2)</td>
</tr>
<tr>
<td></td>
<td>(O_{3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32} = 3)</td>
</tr>
<tr>
<td></td>
<td>(O_{3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32} = 4)</td>
</tr>
</tbody>
</table>

IV. CONCLUSION

The effect of observability of PMU-based measurement infrastructure on the DNR in an active distribution network with stochastic PVs and loads is investigated. For this purpose, a probabilistic DNR model is proposed for minimizing loss and with an additional observability constraint. The simulation results showed that when a few PMUs are employed, the observability constraint might take away the DNR solution from its optimal point. For keeping the network observable during reconfiguration, the number of PMUs should be much greater than when the observability of the network for a single topology is intended.

REFERENCES


